Probability Investigation: SKUNK

In the game of SKUNK, we will roll 2 regular 6-sided dice. Players receive an amount of points equal to the total of the two dice, unless doubles are rolled, then the player scores 0 points.

1. Complete the table to the right that shows how many points are scored based on the total of the dice.

<table>
<thead>
<tr>
<th>Dice 1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>7</td>
<td></td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>6</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What are the most common scores someone could get on a given roll? How often do they occur?

3. The Golden Rule of Probability states that the

\[
P(\text{Event}) = \frac{\text{Number of Success}}{\text{Number of Possibilities}}
\]

This means all probabilities are given as fractions, decimals or percentages.

a. What is the probability that one any given roll in SKUNK that someone will receive a score of 7?

b. A score of 8?

c. What is the probability that someone will receive a score of 7 or 8?

d. Look at your answers for a, b, and c. What can you notice about those two probabilities? This is true whenever we use the word or when speaking about probabilities, as we will see more of later.

4. Consider if we rolled only a single dice.
   a. What is the probability that someone rolled a six?

   b. Now if they add a second dice, what is the probability that they roll a six on the first and a six on the second? You may want to use your table above.

   c. Pay attention to those two probabilities from above. What can you conclude about the use of the word and when discussing probabilities? This is true for all probabilities of independent events as we will see later.

   d. What is the probability that someone rolls doubles? You may want to use the words and and or to do this.
Probability Investigation: Roulette

In the game of Roulette, a ball is spun around the wheel with different slots on it. See the photo to the right for a typical roulette wheel.

There are 18 black sections, 18 red ones and 2 green ones.

1. Wesley bets that the ball will land on Black, what is the probability that he wins?

2. What is the probability that Wesley loses?

3. Wesley winning and losing are called complimentary events because one or the other must happen. What do you notice about their probabilities?
4. When placing bets on Roulette, you place your chips on a Roulette Board that looks like this one to the right. You may be on any of the sections on the bottom. You may also play on a “corner” which covers four different squares, or a “dozen” which is an entire row and marked as “2-1”

Calculate the probability that Jane wins if she bets on...

   a. The “1st 12”

   b. “1-18”

   c. “Even”

   d. The corner that has 20, 21, 23, and 24

   e. The first “dozen”

5. Obviously, it pays better to bet for things with lower odds. Read this link to see about different payouts. [http://www.roulette30.com/2014/04/how-to-play-roulette-beginners-guide.html](http://www.roulette30.com/2014/04/how-to-play-roulette-beginners-guide.html)

So for every $1 you bet on a Corner for example you would receive $8 in payout.

In the game outlined above, Brian and Jane bet $5 each time. How much money would they receive if they win in each of these scenarios?

<table>
<thead>
<tr>
<th>Brian</th>
<th>Payout</th>
<th>Jane</th>
<th>Payout</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>Red</td>
<td>1-18</td>
<td></td>
</tr>
<tr>
<td>1st 12</td>
<td></td>
<td>1-18</td>
<td></td>
</tr>
<tr>
<td>Corner 1,2,4,5</td>
<td>Even</td>
<td>The first dozen</td>
<td></td>
</tr>
</tbody>
</table>
Probability Investigations: Baccarat

The rules of baccarat are pretty simple, but it gets complicated pretty quickly. In the game there is a player and a dealer. When betting, you decide if you want to bet on the player, the dealer, or if you want to bet on a tie.

Both the player and the dealer are given two cards face up. Cards with a number on it are worth the value of their card, Aces are worth 1, and face cards are worth 10. The total score is added up and only the last digit counts as a score. So for example if a player is dealt a 3 and a 4, they have a score of 7, while if someone is dealt a King and a 7, their total of 17 is worth 7 points as well.

Depending on the total score of both the player and the dealer, a third card may be given.

- If a player has a total score of 0-5 they will be given a third card.
- If a player has a total score of 6-7 they will not be given a third card.
- If the player or the banker has a total score of 8-9 neither of them will be given a third card and this will be considered a “natural win”

If a third card is given by the player, the following rules determine if a third card is given to the banker:

- If the banker has a score of 0-2 they take another card
- If they banker has a score of 3-6 they take another card IF they do not have a higher score than the player after the third card.
- If the banker has a score of 7 they must stand

This sounds complicated, but it is pretty easy once we start.

1. Determine the probability that a player will have a score of 8 or 9 on the first two cards.

2. Jimmy was dealt a 2 and a 3, while the dealer was given a King and a 7. What is the probability that Jimmy will win on his third card?
3. If you bet for a tie, you receive 8:1 payout. Meaning if you bet $100 and a tie comes you will receive $800. According to Casino City the probability of a tie happening are around 9.5%. ([http://www.casinocity.com/rule/baccarat.htm](http://www.casinocity.com/rule/baccarat.htm))

In probability the **Expected Value** of a bet is equal to the return multiplied by the probability of it happening.

If you bet $100 on a tie in baccarat, how much would you expect to win?

4. Let’s pretend that the rules change and a player dealt a 6-7 can be given a new card. Jane was dealt a 3 and a 4. If she was given a new card, what is the probability that her score would...
   a. Stay the same?

   b. Increase?

   c. Decrease?
Probability Investigations: Craps

In the game of Craps, players roll two dice and the following happens dependent on their score rolled:
- A 7 or 11 means the player wins
- A 2, 3, or 12 means the player loses
- Any other roll the game continues.

If the game continues, players continue to roll and they will lose if they roll a 7, or they will win if they roll the same score that they got on their first roll. If a different value is rolled, they will continue to roll.

Here are examples of four different players that result in wins or losses:

<table>
<thead>
<tr>
<th>Player</th>
<th>Roll 1</th>
<th>Roll 2</th>
<th>Roll 3</th>
<th>Roll 4</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Player 1</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>WIN</td>
</tr>
<tr>
<td>Player 2</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td>WIN</td>
</tr>
<tr>
<td>Player 3</td>
<td>4</td>
<td>2</td>
<td>12</td>
<td>7</td>
<td>LOSE</td>
</tr>
<tr>
<td>Player 4</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td>LOSE</td>
</tr>
</tbody>
</table>

1. What is the probability that a player wins on the first roll? What is the probability that the player loses? What is the probability that the game continues?
   You may wish to consult this table to the right that we filled out back on our first day of this unit...or not...

2. If a player rolls a 4 and the game continues, what is the chance that they win the game?
   HINT: This seems really hard to figure out, but remember all rolls other than a 4 or 7 do nothing, so just ignore them.
3. This has to do with a concept called **Conditional Probability** and the word “given”. In question 2, we are trying to find the probability that a player wins *given* that they rolled a 4 first.

The rule for conditional probability is \( P(A \cap B) = P(A|B) \cdot P(B) \) where \( P(A|B) \) is the probability of A given B.

Use this formula to determine the probability that a player rolls a 4 on their first roll and wins.

4. Using this same formula, fill out the table to determine the probability that a player wins if the game continues after the first roll.

| Roll | P(Rolled) | P(Win | Rolled) | P(Win and rolled) |
|------|-----------|-------------|------------------|
| 4    |           |             |                  |
| 5    |           |             |                  |
| 6    |           |             |                  |
| 8    |           |             |                  |
| 9    |           |             |                  |
| 10   |           |             |                  |

5. What is the probability that a player wins at craps? *Remember that a player can also win on the first roll (see your answer from question 1)*

6. The payout for craps is even money, meaning if you bet $100 you will win $100. What is the expected value of a $100 bet?
Probability Investigations: Intro to Cards

Each group has been given four cards, don't worry about the names written on them or anything stupid like that. Determine...

1. How many ways you can arrange the four of them in order

2. How many ways you can pick 2 of them where the order matters (King-Queen is NOT Queen-King)

3. How about where order doesn't matter? (King-Queen is the same as Queen-King)

4. You can calculate a lot of this on your calculators by pressing “Math” > “PRB”. There are three specific commands you will want: “!” which represents a Factorial, “nPr” which represents a Permutation and “nCr” which represents a Combination. Try inputting these following commands:
   4!
   4 nPr 2
   4 nCr 2

Compare your answers to this with your answers to questions 1-3. Use this to write a definition for the three bolded terms.
5. In a deck of 52 cards, how many possible hands of 5 are there?

6. How many different ways could the students in this class line up?

7. a) On a locker there are 40 numbers, how many possible 3 number codes are there to open it?

   b) What if the digits can not be repeated?

   c) What if the first number must be even and they can't be repeated?

8. While playing a game of *Words with Friends*, Sarah has the letters: PROBALE. Each letter may only be used once.

   a) How many ways could she rearrange those letters?

   b) How many “words” of three letters could you make? *Note: They don’t have to be actual existing words, just collections of letters.*
c) What is the probability that a three letter “word” has one vowel in it?

d) What is the probability that a three letter “word” has at least one vowel in it?

9. In playing a game of Black Jack, each player is dealt two cards and the object of the game is to total as close to 21 without going over. Face cards are worth 10 and an Ace is worth either 11 or 1, whichever is more beneficial. While playing a game, Sam was dealt a 9/6.
   a) What is the probability that he gets exactly 21 on his next card?

   b) What is the probability that he busts (gets over 21)?

   c) Jennifer is the dealer of this game and she has a Jack showing while her other card is facedown. If Sam stays, what is the probability that he wins? (a tie goes to the dealer)
Probability Investigations: Draw Poker

In the game of Draw Poker, players are dealt five cards and have to make the best poker hand, which are ranked and explained in the table below.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Royal Flush</td>
<td>10-J-Q-K-A all of the same suit</td>
</tr>
<tr>
<td>2</td>
<td>Straight Flush</td>
<td>Five cards in order of the same suit</td>
</tr>
<tr>
<td>3</td>
<td>Four of a Kind</td>
<td>Four card each of the same number</td>
</tr>
<tr>
<td>4</td>
<td>Full House</td>
<td>Three cards of the same number and a pair of a different number</td>
</tr>
<tr>
<td>5</td>
<td>Flush</td>
<td>Five cards of the same suit in any order</td>
</tr>
<tr>
<td>6</td>
<td>Straight</td>
<td>Five card in order, not of the same suit</td>
</tr>
<tr>
<td>7</td>
<td>Three of a Kind</td>
<td>Three cards of the same number</td>
</tr>
<tr>
<td>8</td>
<td>Two Pair</td>
<td>Two sets of pairs</td>
</tr>
<tr>
<td>9</td>
<td>Pair</td>
<td>Two cards of the same number</td>
</tr>
</tbody>
</table>

1. How many different possible hands are there?

2. What is the probability that a player is dealt a Royal Flush?

3. What about a Straight Flush? *Remember, that a Royal Flush does not count as a Straight Flush*
4. What is the probability that you get dealt four Aces? Four of any kind? Remember to take the fifth card in your hand into account.

5. What is the probability that a player is dealt a Full House?

6. Jerri looks at the first two cars that are have been dealt to her. Given that they are both spades, what is the probability that she will have a flush?
7. Given that James was dealt a pair of Aces, what is the probability that he ends up with Two Pairs?

8. Jenny and Johnny are playing. If Jenny is dealt four 5s and the 3 of Spades what is the probability that she wins?
**Probability Investigation: More Dice**

Remember WAAAAAYYYY back to the first page of this packet when we looked at the game SKUNK? Remember: players receive points when the dice are rolled equal to the sum of the pips on both dice. However, if doubles are rolled that player ends up with a score of zero for the round.

Suzie is playing and decides to stand and get all the points she can.

1. What is the probability that she scores points on four rolls before having doubles rolled on the fifth roll?

2. What is the probability that doubles are rolled before the fifth roll?

3. What is the probability that she scores points in more than five rounds? *(HINT: More than five means not five or lower)*

4. What is the probability that Suzie scored points 20 times before rolling doubles on her 21st roll?
5. What is the rule to help Suzie calculate the probability of not rolling doubles until the $n^{th}$ roll?

Ada is going to play a different dice game. She will roll a standard 6-sided die. If she rolls a 6, she wins, otherwise she loses.

6. She plays this game 5 times, what is the probability that she wins exactly 3 times?

7. How many ways could she have won 3 games out of 5?

8. What if she wants to play 10 games, what is the probability that she wins exactly 6 times? *You may want to use your answers to part 6 and 7 to solve this one a little faster.*
9. If Ada plays this game 10 times, what is the probability that she wins 8 or more times?

10. Write down an equation that helps you solve problems like this.

11. The first part of this investigation (Suzie’s game) was called Geometric Probability and Ada’s game is called Binomial Probability. They are both different types of probability distributions when you are repeating independent events (remember two events are independent if the results of one have no impact on the results of the other). What is different about these two scenarios? When should you use Geometric Probability, when should you use Binomial Probability?

On your calculator, you can calculate these two the “distr” command which is the 2nd functions above the “vars” button. It may be helpful to know that pdf stands for “probability density function” which can calculate the probability that exactly \( n \) times will occur. While cdf stands for “cumulative density function”, which is the amount of times \( n \) or fewer times will occur.
Example: A multiple-choice test has five different choices. One particular student did not study and plans on guessing for every question, on a test with 20 multiple choice questions.

12. What is the probability that the student got 10 right?

13. What is the probability that the student got 10 or more correct?

14. The teacher, Mr. Llessur, is feeling very mean. He won’t let his students leave until they get one question right. What is the probability that a student will have to answer 4 questions before they can leave?

15. What is the probability that a student will have to answer 9 or more questions before they can leave?
Appendix Information:

Appendix A: Suggested Unit Schedule- this was compiled for a single, 23 lesson module focused exclusively on probability. Each lesson is 80 minutes long. If your specific teaching situation is different, then you would be encouraged to modify accordingly. Some suggestions are available at The Wandering Math Teacher, the blog of the Author of this work.

Appendix B: Casino Game Investigation – this investigative task was given part way through the course to expose students to different games and explore the probability concepts contained within.

Appendix C: Casino Mathematica Project – this is the cumulative project for this unit. Teachers are encouraged to introduce this very early on in the unit and refer back to it throughout the unit of instruction.

Questions? Comments? Concerns? Please do not hesitate to contact the author on Twitter @RussellMath.
<table>
<thead>
<tr>
<th>Class</th>
<th>Content</th>
<th>Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Introduction to Probability:</strong> Language of probability, simple calculations introduction to compound events</td>
<td>Probability Investigation: SKUNK Pg. 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Probability Investigation: Roulette Pg. 2-3</td>
</tr>
<tr>
<td>2</td>
<td><strong>Compound Events Part 2, expected value.</strong></td>
<td>Probability Investigation: Baccarat Pg. 4-5</td>
</tr>
<tr>
<td>3</td>
<td><strong>Venn Diagrams, and Conditional Probability</strong></td>
<td>Probability Investigation: Craps Pg. 6-7</td>
</tr>
<tr>
<td>4</td>
<td><strong>Quiz Review Day</strong></td>
<td>Fundamentals of Probability Sheet</td>
</tr>
<tr>
<td>5</td>
<td><strong>Quiz on introduction to probability, compound events, and conditional probability</strong></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td><strong>Introduction to Permutations, Combinations, and Factorials</strong></td>
<td>Probability Investigation: Intro to Cards Pg. 8-10</td>
</tr>
<tr>
<td>7</td>
<td><strong>Permutations, Combinations, and Factorials Part 2</strong></td>
<td>Probability Investigation: Draw Poker Pg. 11-13</td>
</tr>
<tr>
<td>8</td>
<td><strong>Review of Counting Principles</strong></td>
<td>Counting Principles Review Sheet</td>
</tr>
<tr>
<td>9</td>
<td><strong>Quiz on Permutations, Combinations, Factorials, and Conditional Probability</strong></td>
<td>Decide on Casino Game to Investigate</td>
</tr>
<tr>
<td>10</td>
<td><strong>Casino Game Investigation Work Day</strong></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td><strong>Casino Game Investigation Work Day</strong></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td><strong>Casino Game Investigation Share Day</strong></td>
<td>Probability Investigations: More Dice (pg. 14-15 #1-5)</td>
</tr>
<tr>
<td></td>
<td>Binomial Probabilities</td>
<td>Geometric Probabilities</td>
</tr>
<tr>
<td>----</td>
<td>------------------------</td>
<td>-------------------------</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Review of Geometric and Binomial Probabilities</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td><strong>Quiz on Geometric and Binomial Probabilities</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Game Design Feedback</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Test Review Day</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td><strong>Cumulative Test Day</strong></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Project Work Day</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Optional Cumulative Test Retake Day/Project Work Day</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Project Work Day</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td><strong>Project Share Day</strong></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td><strong>CASINO DAY~~!!!!!</strong></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>Project Reflection and Course Evaluation</td>
<td></td>
</tr>
</tbody>
</table>
Casino Game Investigation

Individual Assignment.

So far in class, we have looked at many different games of chance including SKUNK, Roulette, Baccarat, Craps, Blackjack, and Draw Poker. Of course, there are dozens more.

Your job is to take an existing game that we have not already looked at in class and analyze the probabilities of it.

Some potential examples include:
- Sic Bo
- Red Dog
- Keno
- Big Six Wheel
- Pai Gow
- Fan-Tan
- Three card poker
- Three Wheel Roulette
- Spanish 21
- Casino War
- Any state lottery

Any other game you may wish to look at or try them at: http://www.vegasslotsonline.com/table-games/

In your analysis, you should include the following information and calculation.
- A brief description of the rules of the game, and the different ways to win and payouts
- A calculation of at least three different probabilities for victories
- An expected value calculation for $100 bets on those three probabilities calculated above
- An example that includes either conditional probability or combinations, permutations, or factorials.
- A conclusion/reflection of some sort: would you recommend someone play this game? If they had to, what would be the smartest (or least dumb...) bet?

You will need to produce a report explaining all of this and share it with the class. The report can take any form you wish including a written report, a PowerPoint/KeyNote presentation, a short film, a website, an infographic, or whatever else interests you.
Casino Mathematica Game Design

Done in Groups of 1-3

The illustrious Casino Mathematica is set to open and wants a new game to attract customers, but also make money. It’s up to you to come up with just such a game, provided that it meets certain criteria.

Your task is to create a game of chance for us to play or study a new one and lead us in a game of this in class. In addition to bringing the game to class. You will need to hand in an analysis of the probabilities. While you can work in a group to create, and run your game, the work must be done individually, and you will all receive individual grades.

The game needs to have the following features:

- Some compound events (AND & OR)
- Combinations, permutations, or factorials need to play a part in the probability OR involve a conditional probability calculation
- Multiple ways to win
- Different payouts for different risks
- You can have the players compete against a dealer (e.g. Blackjack), or against the board (e.g. Roulette).

Your analysis should include:

- A calculation of the probability that the players win in any of the different possible winning scenarios
- The expected return on each bet – you will want to compare this with the actual expected value on different casino games
- An answer to either of the following questions:
  - If someone plays your game 10 times, what is the probability that they win at least 5 times?
  - If someone decided to play the game until they win, what is the probability that they will play the game exactly 5 times?

Your assignment will be due in three stages:

- Share the project with classmates. You should include an explanation of the rules and a demonstration of the calculations. This can take any form you wish including a written report, a PowerPoint/KeyNote Presentation, a short movie, a poster, or whatever else makes you happy
- We will play our casino games in the Student Center with anyone who feels like joining us. Probably during lunch.
- Reflection on the expected outcomes of the game, how much did you win or lose? How did this compare to what you expected to happen?
ACKNOWLEDGEMENTS:

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Questions? Comments? Concerns? Please do not hesitate to contact the author on Twitter @RussellMath.